Heavy Quarkonia from Anisotropic and Isotropic Lattices *

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We report on recent results for the spectrum of heavy quarkonia. Using coarse and anisotropic lattices we achieved an unprecedented control over statistical and systematic errors for higher excited states such as exotic hybrid states. In a parallel study on isotropic lattices we also investigate the effect of two dynamical flavours on the spin structure of charmonium and bottomonium for several symmetric lattices.

1. INTRODUCTION

Heavy quarkonia play an important role for the theoretical understanding of QCD. Their non-relativistic character has frequently been used to perform efficient lattice simulations and has triggered many detailed studies of systematic errors such as lattice spacing artefacts, relativistic corrections and quenching effects. As an additional advantage there exists a wealth of experimental data, which provide an ultimate check on different improvement programmes. Moreover, lattice calculations have also resulted in predictions from first principles for heavy $Q\bar{Q}g$-states containing an additional gluonic excitation [1–3]. However, those attempts were hampered by the rapidly decaying signal-to-noise ratio of such high-energetic hybrid states on conventional lattices.

More recently anisotropic lattices have been used to circumvent this problem by giving the lattice a fine temporal resolution whilst maintaining a coarse discretisation in the spatial direction [4,5]. In a previous study we already reported on first NRQCD results for charmonium and bottomonium hybrid states from anisotropic lattices [6]. In Section 2 we report on further applications of those methods and study also other excitations in heavy quarkonia more carefully.

In quenched simulations without dynamical sea quarks the strong coupling does not run as in nature and therefore one cannot reproduce experimental quantities at all scales. Observed deviations of the quenched hadron spectrum from experiment have been reported previously and an improvement has been noticed once dynamical quarks are inserted into the gluon background [7]. Here we study unquenching effects for heavy quarkonia and report on our results from isotropic lattices in Section 3.

2. EXCITED QUARKONIA

To study excited states with small statistical errors it is mandatory to have a fine resolution in the temporal lattice direction, along which we measure the multi-exponential decay of meson correlators. To this end we employ an anisotropic and spatially coarse gluon action:

$$S = -\beta \sum_{x,\mathrm{i}>\mathrm{j}} \xi^{-1} \left\{ \frac{5}{3} P_{\mathrm{ij}} - \frac{1}{12} \left(R_{\mathrm{ij}} + R_{\mathrm{ji}} \right) \right\} -$$

*talk presented by T. Manke

$$\beta \sum_{x,\mathrm{i}} \xi \left\{ \frac{4}{3} P_{\mathrm{it}} - \frac{1}{12} R_{\mathrm{it}} \right\} \quad . \qquad (1)$$

Here (β, ξ) are two parameters, which determine the gauge coupling and the anisotropy of the lattice. Action (1) is Symanzik-improved and involves plaquette terms, $P_{\mu\nu}$, as well as rectangles, $R_{\mu\nu}$. It is designed to be accurate up to $\mathcal{O}(a_s^4, a_t^2)$, classically. To reduce the radiative corrections we used self-consistent mean-field improvement for both spatial and temporal links. With this prescription we expect only small deviations of ξ from its tree-level value a_s/a_t.

For the heavy quark propagation in the gluon background we used the NRQCD approach on anisotropic lattices as described in [8]. From the quark propagators we construct meson correlators for bound states with quantum numbers $S(0,1) \times L(0,1,2)$ and for hybrid states with additional gluonic excitation. For example, the spin-singlet operators read

$$\bar{Q}^\dagger Q \ , \ \bar{Q}^\dagger \Delta_i Q \ , \ \bar{Q}^\dagger \Delta_j \Delta_k Q \ \text{ and } \ \bar{Q}^\dagger B_i Q \ . \qquad (2)$$

Within the NRQCD approach it is paramount to establish a scaling region at finite lattice spacing. Our results in Fig. 1 demonstrate that we succeeded in finding such a window. As we only measure excitation energies relative to the ground state it is natural to present our results as the ratio $R_X = (X - 1S)/(1P - 1S)$, which gives the normalized splitting of state X above the 1S.

For the lowest lying hybrid excitations, $c\bar{c}g$ and $b\bar{b}g$, our results from leading order NRQCD [6] are in excellent agreement with previous calculations on isotropic lattices [2,3], but with much smaller errors. This is the combined success of anisotropic and coarse lattices with a clear signal over many timeslices at small computational cost. Here we have also checked the spin-averaged hybrid against possible finite volume errors, temporal lattice spacing artefacts and relativistic corrections, but we did not find any significant effect.

In Fig. 1 we have also shown our new results for higher radial excitations and D-states ($L = 2$). Since all the spin corrections up to $\mathcal{O}(mv^6)$ are now included in our analysis we can also determine the spin-structure very accurately. In particular we were able to extract the exotic hybrid quarkonia, ${}^3B_1^{-+}$, explicitly for the first time from NRQCD. Our data indicates that the spin splittings in hybrid states are enlarged compared to P-states, whereas the D-state splittings are much suppressed. A more detailed discussion of the spin structure in quarkonia is presented elsewhere [9].

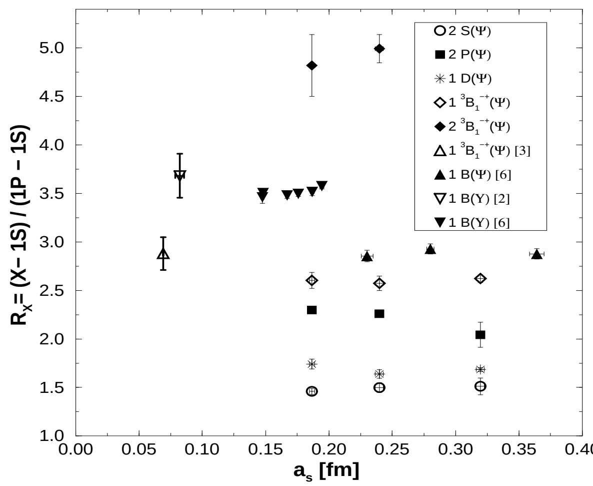

Figure 1. Scaling analysis for excited Quarkonia. We plot the ratio R_X against the spatial lattice spacing for different states X = 2S,2P,1D and magnetic hybrids (1B,2B).

The dominating systematic error for all the predictions in this section is an uncertainty in the scale as a result of the quenched approximation. This is not yet controlled and we find a variation of 10-20%, depending on which experimental quantity is used to set the scale.

3. SPIN STRUCTURE AND SEA-QUARK EFFECTS

To study sea quark effects in heavy quarkonia we employ an ensemble of isotropic lattices, where the gauge configurations were generated from an RG improved gluon action and tadpole-improved SW fermions for two flavours of light sea-quarks [7]. We then performed an NRQCD calculation of S- and P-states at two different gauge couplings of $\beta = 1.80$ and $\beta = 1.95$, corresponding to $a \approx 0.2$ fm and $a \approx 0.15$ fm. In addition to the chiral extrapolation from four different sea quark masses,

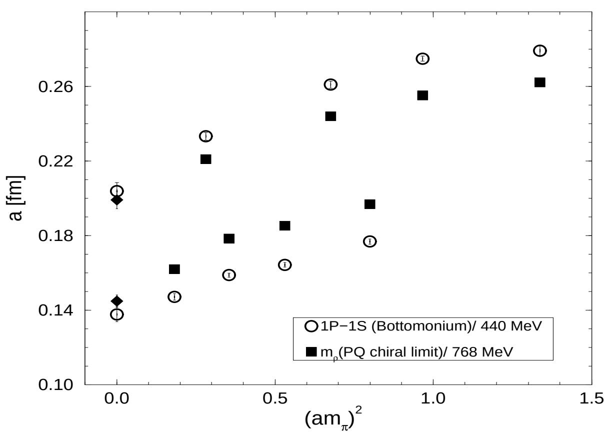

Figure 2. Chiral extrapolation of the lattice spacing at two different couplings determined from m_ρ (squares) and $1P-1S$ in Bottomonium (circles).

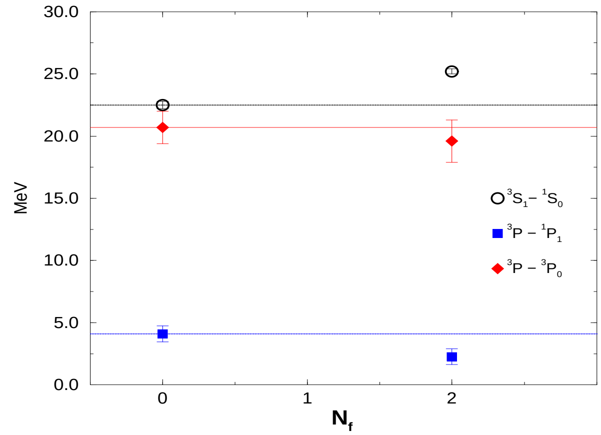

Figure 3. Spin structure of Bottomonium. Here we plot results from quenched and two-flavour QCD at the same lattice spacing $a \approx 0.164$ fm.

we also compared our results directly to quenched simulations at the same lattice spacing.

In Fig. 2 we see an encouraging trend for the lattice spacings from different physical quantities to agree much better in the chiral limit. This is an improvement over quenched simulations, where one has large uncertainties in the scale.

Furthermore, we observed a clear shift upwards of the hyperfine splittings as the sea quark mass is decreased. For the Bottomonium we find an effect of about 3 MeV in a direct comparison to a quenched calculation using an identical formulation of tadpole-improved NRQCD with accuracy $\mathcal{O}(mv^6)$. This is a 10-σ effect (or 10%) as shown in Fig. 3. In Charmonium the hyperfine splitting is also raised by about +15%, i.e. to around 60 MeV in the chiral limit on our coarsest lattice.

In P-states one expects a different situation, since their wavefunctions vanish at the origin. Indeed, the very small hyperfine splitting (${^3P}-{^1P_1}$) is further suppressed ($\approx$ 3-σ) on dynamical configurations and there is no resolvable shift for the fine structure, e.g. ${^3P} - {^3P_0}$. This validates the quenched approximation for such quantities.

For the Bottomonium system we are presently performing a similar analysis at $\beta = 2.1$ ($a \approx 0.1$ fm) to determine whether those observations still hold on finer lattices.

This work is supported in part by Grants-in-Aid of the Ministry of Education, Science and Culture (Nos. 09304029, 10640246, 10640248, 10740107, 11640250, 11640294, 11740162). SE and KN are JSPS Research Fellows. AAK, HPS and TM are supported by the Research for the Future Program of JSPS, and HPS also by the Leverhulme foundation.

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